**Embedded Options in Insurance Products**

# Variable Annuities

* Traditional Annuities are savings vehicles in which premiums paid by the policyholder are put into an account that grows at a **fixed rate of interest (Account Value will only grow)**
* Variable Annuities are similar, just that the premiums are invested in some underlying assets, which means that it **grows at the investment yield (Account Value can fluctuate)**
* After the accumulation period, the amount in the account is used to **purchase an annuity**, which provides a **fixed stream of payments** for a set amount of time
* In order to make variable annuities more appealing, insurance companies usually have **guarantees** on some of the benefits provided
  + These guarantee provide a minimum amount on the benefits provided
  + Thus, the actual benefits provided is the higher of the actual values and the guarantees
  + This forms a **maximum function** which can be expressed in the **form of an Embedded Option**

|  |  |
| --- | --- |
| **Minimum Death Benefit (GMDB)** | Minimum **amount paid** to beneficiary on policyholder death |
| **Minimum Accumulation Benefit (GMAB)** | Minimum **account value** after some period of time |
| **Minimum Withdrawal Benefit (GMWB)** | Minimum **withdrawal amount** on the annuity purchased |
| **Minimum Income Benefit (GMIB)** | Minimum **purchase price** of the annuity (& hence income) |

## Guaranteed Minimum Death Benefit (GMDB)

* Also known as a **Return of Premium Guarantee** - Death Benefit will always be the higher of the Account Value or Amount Invested
* As shown below, it can be expressed in the form of a **Put Option** with a time to maturity equal to when the policyholder dies (**Life Contingent**)
* Using the **life distribution** of the individual, we can find the Expected Value of the Put Option
  + If there are a large number of similar policyholders, then in aggregate the Death of policyholders should follow a **Mortality Distribution**
  + Thus, the insurer can be more certain of the number of Put Options exercised in each period
  + The expected value can be then interpreted as the **average cost per contract** of the guarantee



$$\text{GMDB} = \max(S\_T, K)$$

$$\text{GMDB} = \max(S\_T, K) – S\_T + S\_T$$

$$\text{GMDB} = \max(0, K – S\_T) + S\_T$$

$$\text{GMDB} = \text{Put Option on Death} + \text{Account Value}$$









* Note that it can be manipulated to be a **Call Option** instead with the guaranteed amount

$$\text{Expected Value of Put} = \int^m\_0 P(t) \cdot f\_{T\_x} \mathrm{d}t$$

$$\text{Expected Value of Put} = P(t) \cdot \int^m\_0 f\_{T\_x} \mathrm{d}t$$

$$\text{Expected Value of Put} = \text{Put Value} \cdot \text{Probability of Death by time } m$$







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## Earnings Enhanced Death Benefits (Rider)

* Feature that pays an **additional amount** on top of the Death Benefit equal to the excess of the Account Value over the initial amount invested
* Similarly, it can be expressed in the form of a **Call Option** with a time to maturity equal to then the policyholder dies (**Life Contingent**)
* Using the **life distribution** of the individual, we can find the Expected Value of the Call Option



$$\text{Earnings Enhanced Benefit} = x\% \cdot \max(S\_T – K, 0)$$

$$\text{Earnings Enhanced Benefit} = x\% \cdot \text{Call Option on Death}$$





$$\text{Expected Value of Call} = \int^{\infty}\_0 C(t) \cdot f\_{T\_x} \mathrm{d}t$$

$$\text{Expected Value of Call} = C(t) \cdot \int^{\infty}\_0 f\_{T\_x} \mathrm{d}t$$

$$\text{Expected Value of Call} = \text{Call Value} \cdot \text{Probability of Death by time } m$$







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Note that the guarantee may have different effects in different periods:

$$0 – m\_1 \rightarrow x\%$$

$$m\_1 – m\_1 \rightarrow y\%$$





This can be expressed in one of two ways:

|  |  |
| --- | --- |
| **Standard Method** | **Alternative Method** |
|  |  |

## Guaranteed Minimum Accumulation Benefit (GMAB)

* Identical to the Minimum Benefit case, but is instead contingent on the **policyholder surviving**
* Thus, the expected value of the Put is the



$$\text{GMAB} = \max(S\_T, K)$$

$$\text{GMAB} = \max(S\_T, K) – S\_T + S\_T$$

$$\text{GMAB} = \max(0, K – S\_T) + S\_T$$

$$\text{GMAB} = \text{Put Option on Maturity} + \text{Account Value}$$









$$\text{Expected Value of Put} = P(m) \cdot P(T\_x \geq m)$$

$$\text{Expected Vlaue of Put} = \text{Put Value} \cdot \text{Probability of Survival by time } m$$





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### Crash Course on Life Contingencies

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## Other Guarantee Features

* **Guarantee Formulas** - Option to recalculate the guarantees at the current conditions; Exercised when the current conditions are deemed to be favourable
* **Underlying Asset** - Option to change the underlying assets; Exercised when the current assets are not expected to perform well
* **Contract Termination** - Option to terminate the policy early to obtain the account value; Exercised when the account value is significantly larger than payout

# Mortgage Guarantee Insurance

* When purchasing a house, most people would have to take up a loan to do so - **Mortgage**
* The house is often used as Collateral which will be repossessed by the Lender in the event of the default to be sold to cover the loan amount
* However, the sale of the house may not always be able to cover the amount, thus there is always some **risk to the lender**
* Thus, the Lender can cover their risk by purchasing **Mortgage Guarantee Insurance**
* This helps Lenders to manage their Credit Risk - which **increases Liquidity** throughout the market



$$\text{Loss = \max(B + C – R, 0)$$



Thus, the lender has a **short position in the Put Option** while the Borrower has a long position. The borrower will exercise the option and default is the value of the home is significantly lower than the mortgage. However, they will face consequences such as a **lower credit score**

# Property Insurance

* Indemnifies the cost of damaged property, **up to the sum insured**
* However, the Cost of the damaged property may end up being much larger than the sum insured
* Thus, policyholders can add on a rider that **guarantees that the cost of damage will be covered**



$$C = \min(C, I) + \max(C – I, 0)$$



# Pension Plans

**Full indexing, partial indexing**

* Guarantees that the payments will never have less purchasing power than the first payment
* Guarantees that the size of payments will never decrease

**Risk Management in Insurance Products**

# Types of Hedging

|  |  |
| --- | --- |
| **Static Hedging** | **Dynamic Hedging** |
| Based on **Option Payoff** | Based on **Option Value** |
| Held to **Expiration** | Frequent **Trading** |
| Frequently Reviewed | Complicated & Expensive |

## Hedging Embedded Options in Guarantees

* They tend to be extremely complex, thus regular Calls and Puts are often insufficient to fully hedge them
* **Exotic Options** are used instead - which are non-standard options available OTC which means that they are **customized**

|  |  |  |
| --- | --- | --- |
| **Exotic Option** | **Brief Description** | **Relationship** |
| **Forward Start** | Pay for option ahead of time | Policies with guarantees that **start only in the future** |
| **Chooser** | Choose type of option | Policies with **different possible guarantees** |
| **Lookback** | Value based on historical performance | Policies whose guarantees are periodically recalculated **relative to the past values** |
| **Shout** | Shout to set minimum at time | Policies where policyholder **decides time** of calculation of the guaranteed benefits |
| **Rainbow** | Dependent on multiple assets | Policies based on **multiple assets** |

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## Hedging Catastrophe Risk

* When Insuring individual properties or lives, these events tend to be treated independently
* However, in the event of natural disasters, all homes or lives within a region will be affected
* Insurers typically dealt with this risk through Reinsurance, but they have recently begun transferring the risk to the financial markets instead through derivatives:
  + **Weather Derivative** → Contingent on weather event not happening
  + **Catastrophe Bond** → Contingent on catastrophe occurring not happening
  + These derivatives are usually issued by a **Special Purpose Vehicle** (related to the insurer)